Non-stop outdoor navigation of a mobile robot
- Retroactive positioning data fusion with a time consuming sensor system -

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Abstract
We propose a position estimation technique for non-stop outdoor navigation of an autonomous mobile robot. The proposed position estimation technique is based on maximum likelihood estimation. To cope with the parallel processing of internal and external sensor information and time delay in the sensor data process, we introduce the retroactive positioning data fusion technique. The proposed technique is implemented on our small size autonomous mobile robot. An experimental result is shown, in which our robot could navigate itself without stopping even when it takes several seconds of processing time to detect landmark from external sensor data.

1 Introduction
“Where is the robot now?” is one of the most important subjects for an autonomous mobile robot. There are many research about the technique of exact current position estimation of the robot[1] ~ [4]. In this paper, we discuss “When robot knows its past position retrospectively, what should the current position and the uncertainty be?” Here, we suppose that the robot moves in two dimensional work space, and we define the word position as the robot’s location and its heading direction.

Let us suppose the case that a person walks in the street and he wonders his current location. He happens to walk across a corner, which he does not remember instantly where it is. In this situation, he does not know his exact location until he remembers the place of the corner. But, he does not need to stop while he wonders where the corner is, because he can walk while considering it and may get more information about his current location while walking. When he remembers the place of the corner he walked across, he understands his past position exactly and can estimate his current position if he can memorize his motion while walking.

When the mobile robot navigates on the two dimensional plane; it uses its sensor such as vision system to know its position, besides the dead reckoning system(odometry system etc.). In this case, the processing time of sensor data is common problem and often it takes more than few seconds. When the robot hopes to move continuously even while it processes the vision sensor information for detecting special landmarks, the robot is in the similar situation as above example of a person. When the robot gets its position by the result of landmark observation, it knows its past position exactly. Therefore, the technique for a retroactive position estimation and re-estimation of current position are required. If the robot has such ability, it can move faster because it can continue to move while it processes to find the landmark to know its exact position.

In this paper, we propose the position estimation technique for a mobile robot without the needs to stop even when it takes several seconds to detect landmark by the external sensor. We also demonstrate its usefulness by showing the experimental results of outdoor navigation using the practical robot with proposed method. Especially, our method is useful for the small size autonomous mobile robot which has a limited processing power.

2 Previous work and problem
We have developed the technology to navigate an autonomous mobile robot long distance robustly in outdoor environment. Supposing that the robot has environment map and path to be followed in advance, we have proposed the method to navigate robot autonomously from starting point to goal point and have shown some experimental results. Our basic strategy for autonomous navigation of mobile robot is to follow the given path estimating the robot position exactly. When robot knows its initial position, the robot can calculate its position through accumulating wheel rotations. We call this method dead reckoning. But, by only dead reckoning, it is impossible to navigate long distance because the error of position by dead reckoning are accumulating gradually. Therefore, robot must verify its position by the observation of special landmark which is included in information of environment map.

In our method, robot always calculates not only
its position but also its covariances. When the landmark is detected, the robot corrects its position by fusion of current position, its covariances and the position obtained from landmark observation based on maximum likelihood estimation. Strong point of this method is that robot can corrects all parameters of its three dimension position \((x, y, \theta)\) even when the position obtained from landmark observation is one or two dimensional information\([2, 6]\). Because covariances have not only the magnitude of the error of position but also correlation between each parameter of position. We also confirmed the great usefulness of this method in the experiment of real environment using real robot\([5, 6]\).

But, this method has a weak point which is that robot must stop in case that the processing time of landmark information can not be ignored. If robot moves while processing landmark information, the position obtained from landmark information becomes it of past time. Our previous proposed method can not treat this case, because robot has always only current position and its covariances. Kosaka et al. were also in the face of same problem in their research which is vision based indoor navigation of a mobile robot using extended Kalman filtering. They proposed a retroactive updating of positional uncertainty based on command history executed while the robot is processing the image\([7]\). Their robot could navigate 10.3m/min without stopping to process the image by their proposed method. The aim of our work is similar. But, in this paper, we propose more simple and general way for positional uncertainty estimation, which re-calculates the estimated position and its uncertainty from the increments of these values obtained by dead reckoning. This method is also extended to the case of the parallel processing of multiple kinds of landmark observation in the position estimation.

3 Proposal of retroactive positioning data fusion and recalculation of current robot position

Figure 1 shows the case that it takes \(n\tau\) seconds to process the landmark information. At first, robot observes a special landmark to correct its position at time \(t_0\)(left side in Figure 1). Next, robot continues to move while processing landmark information and the error of robot position become larger gradually (center in Figure 1). At last, at time \(t_0 + n\tau\), robot gets new information about its position at time \(t_0\) from the result of the process of landmark information(right side in Figure 1). But, in previous our method, it is fundamentally impossible to correct robot position using this new information, because the robot has only position and covariances at time \(t_0 + n\tau\). Therefore, robot must have more information to correct its position at time \(t_0\) and recalculate current position.

At first, we propose that robot keeps the position and the covariances at sensing time \(t_0\) for retroactive positioning data fusion. So, when the process of landmark information finishes at time \(t_0 + n\tau\), the position at time \(t_0\) is corrected easily based on the maximum likelihood estimation retrospectively. Then, if the robot keeps all internal sensor data for the calculation of dead reckoning from the time \(t_0\) to time \(t_0 + n\tau\), it can calculate its current position again after the robot position at time \(t_0\) is corrected based on the landmark information by the external sensor. But, such calculation needs much data and heavy calculation. Therefore, we propose the simple technique for recalculation of current position using only the total increment of parameters such as location, heading and the covariance from the time \(t_0\) to the current time \(t_0 + n\tau\).

In next section, we derive what parameters should be kept for the purpose to minimize the amount of the recorded data and the time of recalculation.

4 Analysis of parameters recorded for recalculation

In this section, we analyze what parameters of dead reckoning should be recorded for recalculation and derive how to recalculate using these parameters. At first, we describe the robot position and uncertainty estimation by dead reckoning. Second, we describe the correction of robot position by the observation of landmark based on maximum likelihood estimation. At last, the formula of current position estimation is derived when the past position is corrected.

4.1 The robot position and the uncertainty estimation by dead reckoning

We derive the relationship of the position and the uncertainty separated \(n\) sampling periods by dead reckoning for the recalculation from time \(t_0\) to \(t_0 + n\tau\). At first, we describe the formula to update the robot position and the uncertainty at every sampling based on dead reckoning. Second, we extend this updating formula to express the relationship of \(n\) sampling periods separated data.

4.1.1 Updating the robot position and the uncertainty at every sampling period

We represent robot’s position and the uncertainty at every sampling period as

\[
P[t] = \begin{bmatrix} x(t) & y(t) & \theta(t) \end{bmatrix}^T
\]

where \((x(t), y(t))\) is the two dimensional location of robot and \(\theta(t)\) is the robot’s orientation. The magnitude of estimated errors are represented by the covariance matrix of \(\Sigma_p[t]\)

These variables are updated every sampling interval \(\tau\) by accumulating displacement given from rotation of the robot’s wheels, as

\[
P[t + \tau] = P[t] + \tau \begin{bmatrix} v[t]\cos(\theta[t]) \\ v[t]\sin(\theta[t]) \\ \omega[t] \end{bmatrix} + \tau n[t](1)
\]

Where, \(v[t]\) is the velocity, \(\omega[t]\) is the rotational angular velocity of robot’s body, \(n[t]\) includes errors of calculations and sampling.

Let the wheels velocity and the trend be expressed in the vector \(m[t]\). And, \(f(p[t], m[t])\) represents first
and second terms of equation (1). $\hat{P}[t]$ is the estimated value of $P[t]$, $\Delta P[t]$ is the errors of $\hat{P}[t]$, $\hat{m}[t]$ is the measured value of $m[t]$, and $\Delta m[t]$ is the errors of $\hat{m}[t]$. Then,

$$
P[t + \tau] = f[P[t], m[t]] + \tau n[t]$$

$$= f[\hat{P}[t] + \Delta P[t], \hat{m}[t] + \Delta m[t]] + \tau n[t]$$

$$\approx f[\hat{P}[t], \hat{m}[t]] + j[t] \Delta P[t] + k[t] \Delta m[t] + \tau n[t]$$

$$= \hat{P}[t + \tau] + \Delta P[t + \tau]$$

Therefore, the errors in odometry increase as

$$\Delta P[t + \tau] = j[t] \Delta P[t] + k[t] \Delta m[t] + \tau n[t]$$

(3)

Where,

$$j(t) = \frac{\partial f(P[t], m(t))}{\partial P(t)} \hat{P}(t), \hat{m}(t)$$

$$k(t) = \frac{\partial f(P[t], m(t))}{\partial m(t)} \hat{P}(t), \hat{m}(t)$$

(4)

The covariance matrix $\Sigma P[t]$ is represented as

$$\Sigma P[t] = E(\Delta P[t] \Delta P[t]^T)$$

(5)

Therefore, with sampling interval $\tau$, the covariance matrix $\Sigma P[t]$ is updated as

$$\Sigma P[t + \tau] = j \Sigma P[t] j^T + k \Sigma m k^T + \tau^2 \Sigma N$$

(6)

Where, $\Sigma m$ is a covariance matrix of $\Delta m$ and $\Sigma N$ is a covariance matrix of $n$.

### 4.1.2 Relation between the position data and the covariance matrices separated $n$ sampling periods

We derive the relation of the position and the covariance matrix separated $n$ sampling periods from (1) and (6) step by step.

The robot position is

$$\hat{P}[t_0 + n \tau] = \hat{P}[t_0] + \hat{P}_n$$

(7)

Where, $\hat{P}_n$ is displacement during $n$ sampling periods.

And, the covariance matrix is

$$\Sigma P[t_0 + n \tau] = J_n \Sigma P[t_0] J_n^T + K_n + N_n$$

(8)

Where,

$$J_n = j_{n-1} j_{n-2} \cdots j_0$$

(9)

$$K_n = k_{n-1} \Sigma m k_{n-1}^T + j_{n-1} k_{n-2} \Sigma m k_{n-2}^T j_{n-1}^T + \cdots + j_0 k_0 \Sigma m k_0^T j_0^T$$

(10)

$$N_n = \tau^2 \Sigma n + j_{n-1} \tau^2 \Sigma m j_{n-1}^T + \cdots + j_0 \tau^2 \Sigma m j_0^T$$

(11)

and $j_i$ means $j[t_0 + i \tau]$ and $k_i$ means $k[t_0 + i \tau]$.

### 4.2 Correction of robot position by the observation of landmark

Here, we describe the formulation of the correction of robot position at time $t_0$ based on the result of the observation of landmark when the process of landmark observation finished at time $t_0 + n \tau$. Here, we denote the robot position at time $t_i$ estimated at time $t_j$ by $\hat{P}[t_i | t_j]$. Generally, if the external sensor does not need
4.2.1 Representation of position information obtained by landmark observation

We define the vector of information obtained from the observation of a landmark as

\[ s = [s_1, s_2, \ldots, s_n]^T \]  

Then, the covariance matrix is \( \Sigma_s \).

The information provided by a landmark is represented as

\[ g(P[t_0], s) = 0 \text{ for } s \neq 1 \]  

We call these equations Constraint equations.

Supposing that the errors in position \( \hat{P}[t_0|t_0] \) estimated by dead reckoning are small, we can linearize (13) around \( P[t_0|t_0] \) and get

\[ g(\hat{P}[t_0], \hat{s}) + J_p(P[t_0] - \hat{P}[t_0|t_0]) + J_s \Delta s = 0 \text{ for } s \neq 1 \]  

Where, \( \hat{s} \) is the measured value of \( s \) and \( J_p \) and \( J_s \) are given as

\[ J_p = \frac{\partial g(P[t_0], s)}{\partial P[t_0]} \Bigg|_{\hat{P}[t_0], \hat{s}} \quad J_s = \frac{\partial g(P[t_0], s)}{\partial s} \Bigg|_{\hat{P}[t_0], \hat{s}} \]  

By normalizing both sides of (14) by each row vector of \( J_p \) (14) becomes

\[ J_p(P[t_0] - \hat{P}[t_0|t_0]) = -G - J_s \Delta s \]  

Then, the sensor information obtained from tree detection sensor is represented as

\[ P_{su} = J_p(\hat{P}_s - \hat{P}[t_0|t_0]) \]  

And the covariance matrix \( \Sigma_{su} \) is given as

\[ \Sigma_{su} = J_s \Sigma_s J_s^T \]

4.2.2 Position correction by Maximum Likelihood Estimation

To fuse the information about robot position obtained from the landmark observation with the estimated dead reckoning vector \( \hat{P}[t_0|t_0] \), the vector \( \hat{P}[t_0|t_0] \) is transformed into the same coordinate as (16).

\[ \hat{P}_s[t_0|t_0] \] denotes the vector \( \hat{P}[t_0|t_0] \) in converted coordinates and \( \Sigma_{su}[t_0|t_0] \) is its covariance matrix.

\[ \hat{P}_s[t_0|t_0] = 0 \]  

\[ \Sigma_{su}[t_0|t_0] = J_p \Sigma_p[\hat{P}_s|t_0] J_p^T \]  

Then, Maximum Likelihood Estimation is formulated as

\[ \Sigma_{fu} = (\Sigma_{u}[t_0|t_0]^{-1} + \Sigma_{su}^{-1})^{-1} \]

\[ \hat{P}_f[u] = \Sigma_{fu} \Sigma_{su}^{-1} \hat{P}_s \]  

Equations (21) (22) are transformed back into the expression of \( x,y,\theta \) axes as,

\[ \Sigma_f[t_0|t_0 + n\tau] = J_p^{-1} \Sigma_{fu}(J_p^{-1})^T \]

\[ \hat{P}_f[t_0|t_0 + n\tau] = \hat{P}[t_0|t_0] + J_p^{-1} \hat{P}_f \]

This formulation fuses position information from landmark observation and position information by dead reckoning to obtain a corrected estimated position. This formulation is used in case that the dimension \( m \) of constraint equations is less than three [2][6].

4.3 The current position and the uncertainty estimation by recalculation

At last, when the robot position at time \( t_0 \) is corrected based on the formula given above subsection, we derive the formula of recalculation from time \( t_0 \) to time \( t_0 + n\tau \).

The terms \( \hat{P}_n, \Sigma_n, \Theta_n, \hat{N}_n \) in equations (7) and (8) depend on the heading of robot \( \theta \). Therefore, these terms must be modified when \( \theta \) at time \( t_0 \) is corrected to \( \theta + \alpha \).

The modification of these terms is obtained using rotation matrix

\[ R(\alpha) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

At first, \( \hat{P}_n \) is modified as \( \hat{P}'_n = R(\alpha) \hat{P}_n \). In next, \( J'_n \) after modifying \( J_n \) is obtained from replacing third column vector \( J_{n3} \) of \( J_n \) with \( R(\alpha) J_{n3} \). Then, \( K_n \) is modified as \( K'_n = R(\alpha) K_n R(\alpha)^T \). \( N_n \) can not modified using \( R(\alpha) \). But, \( N_n \) is very small. Therefore, we can neglect it to be \( N'_n \simeq R(\alpha) N_n R(\alpha)^T \). Consequently, when \( \hat{P}[t_0|t_0] \) is modified to \( \hat{P}'[t_0|t_0 + n\tau] \) and \( \Sigma'[t_0|t_0] \) is modified to \( \Sigma'[t_0|t_0 + n\tau] \). Formula to obtain \( \hat{P}'[t_0 + n\tau|t_0 + n\tau] \) is given as

\[ \hat{P}[t_0 + n\tau|t_0 + n\tau] = \hat{P}[t_0|t_0 + n\tau] + \hat{P}'[t_0|t_0 + n\tau] \]

and formula to obtain \( \Sigma[t_0 + n\tau|t_0 + n\tau] \) is given as

\[ \Sigma'[t_0 + n\tau|t_0 + n\tau] = J'_n \Sigma'[t_0|t_0 + n\tau] J'^T_n + K'_n \]

Next, the data to be kept for the retroactive positioning data fusion and recalculation is only four data

\[ \hat{P}[t_0|t_0], \Sigma_{t_0|t_0}, \hat{P}_n, \Sigma_{n} \]

(27)
5 Extension to parallel processing of multiple landmarks observation

In this section, we discuss the case of parallel processing of multiple landmarks information which is gotten by several external sensors. If the processes of multiple landmarks information do not overlap on time axis, algorithm for retroactive positioning data fusion is the same as one landmark. But, if the processes of multiple landmarks information overlap, we must consider the order of correction and recalculation to calculate the current robot position consistently. For example, let us suppose the processes of two landmarks information overlap on time axis. In this case, two patterns of overlapping can be considered as Figure 2. CASE 1 shows that two landmarks sensing overlap partly. When the process of landmarks sensing $S_j$ finishes at time $T_j'$ and corrects its position of time $T_j$, the robot position of time $T_i'$ later than time $T_j$ was already corrected. CASE 2 shows that two landmarks sensing overlap completely. When the process of landmarks sensing $S_i$ finishes at time $T_i'$ and corrects its position at time $T_i$. The robot position of time $T_i'$ later than time $T_i$ was already corrected. These problems can be solved easily by that the recorded positioning data of time $T_j$ is also corrected when the processing result of landmark sensing $S_i$ is given at time $T_i'$. This algorithm is the same in case of overlapping the processes of more than two landmarks information.

But, the recorded positioning data should manage carefully. We call retroactive records the set of data for retroactive positioning data fusion and recalculation. It is convenient that retroactive records manage as the linear list. The retroactive record is always added behind the last records because the real time does not reverse. The retroactive record is removed in two case. One is that the landmark is not detected from external sensor information. Another is that the retroactive record became first record and the current robot position is already corrected by the record.

6 Experiment

We implemented on the proposed position estimation algorithm on our experimental mobile robot. Several experiments were performed in the environment which includes the paved road lined with trees and the tiled road with trees and hedges (Figure 3). One of the results of the experiment is shown here. The robot runs the straight 43 meters long using tree landmarks and turns to the right and runs the straight 30 meters long using tree and hedge landmark. The hedge is detected by ultrasonic sensor immediately. But, it takes about 2 seconds to detect the tree using our tree detection sensor SONAVIS[6] which combines sonar and vision. Therefore, our previous robot must have stopped on each sensing point in the experiment[6]. In this experiment, the robot could robustly navigate itself without stopping by detecting landmark and updating its own position. The speed of robot is 25cm/s. Figure 4 is the synthesized photograph of the trajectory of the robot on this experiment.

Figure 5 is a simulation result of the correction of robot position using tree and hedge landmarks. Robot corrects its position using hedge landmarks while processing tree landmark information. When the process of tree landmark information finishes, robot decreases the uncertainty of its position and gets more reliable position. In above experiment, robot performed the same calculation as this simulation.

![Figure 3: Environment of the experiment of non-stop self-guidance of a mobile robot using trees and hedges](image-url)
Figure 4: Trajectory of a mobile robot on the experiment of non-stop self-guidance using trees and hedges.

7 Conclusions

We proposed the position estimation technique for non-stop outdoor navigation of an autonomous mobile robot based on retroactive positioning data fusion and recalculation using increment of robot position vector and covariance matrix obtained by dead reckoning. Of course, in our technique, the robot position must be corrected before robot lost its own way or went out from road area. For this purpose, the estimated uncertainty of the current robot position should be observed in total behavior control level. In future work, we will achieve half-automatic map building by teaching the path and play back autonomous navigation, which realize easily the long distance navigation.

References


